> # Exercice 1

> restart : t1, t2, t3, t4 := solve($X^4 - X^2 + 3$)

$$t1, t2, t3, t4 := \frac{\sqrt{2-2\,\mathrm{I}\sqrt{11}}}{2}, -\frac{\sqrt{2-2\,\mathrm{I}\sqrt{11}}}{2}, \frac{\sqrt{2+2\,\mathrm{I}\sqrt{11}}}{2}, -\frac{\sqrt{2+2\,\mathrm{I}\sqrt{11}}}{2} \quad (1)$$

> $t := [t1, t2, t3, t4] : sum\left(\frac{t[i]^3}{t[i]^4 - 1}, i = 1..4\right)$

$$0 \quad (2)$$

> # Exercice 2

> restart

> $P := x^{2\cdot n} - n^2\cdot x^{n+1} + (2\cdot n^2 - 2)\cdot x^n - n^2\cdot x^{n-1} + 1$

$$P := x^{2n} - n^2 x^{n+1} + (2n^2 - 2) x^n - n^2 x^{n-1} + 1 \quad (3)$$

```
> while simplify(subs(x = 1, P)) = 0 do
   P := simplify(diff(P, x));
  od: P
```

$$-\left(\left(-16n^2 + 32n - 12\right)x^{2n-4} + \left(\left(-2n^3 + 6n^2 + 2n - 6\right)x^{n-4} + \left(\left(n^2 - 7n + 12\right)x^{n-5} + n\,x^{n-3}\,(n+1)\right)n\right)(n-2)\right)n\,(n-1) \quad (4)$$

> # À la fin de la boucle, le degré du polynôme a chuté de 4, donc la multiplicité de 1 comme racine est 4

> $n := 20; simplify\left(\frac{x^{2n} - n^2 x^{n+1} + (2n^2 - 2)x^n - n^2 x^{n-1} + 1}{(x-1)^4}\right)$

$$n := 20$$

$$x^{36} + 4x^{35} + 10x^{34} + 20x^{33} + 35x^{32} + 56x^{31} + 84x^{30} + 120x^{29} + 165x^{28} + 220x^{27} + 286x^{26} + 364x^{25} + 455x^{24} + 560x^{23} + 680x^{22} + 816x^{21} + 969x^{20} + 1140x^{19} + 1330x^{18} + 1140x^{17} + 969x^{16} + 816x^{15} + 680x^{14} + 560x^{13} + 455x^{12} + 364x^{11} + 286x^{10} + 220x^9 + 165x^8 + 120x^7 + 84x^6 + 56x^5 + 35x^4 + 20x^3 + 10x^2 + 4x + 1 \quad (5)$$

> $factor\big((x-1)^4\cdot(x^{36} + 4x^{35} + 10x^{34} + 20x^{33} + 35x^{32} + 56x^{31} + 84x^{30} + 120x^{29} + 165x^{28} + 220x^{27} + 286x^{26} + 364x^{25} + 455x^{24} + 560x^{23} + 680x^{22} + 816x^{21} + 969x^{20} + 1140x^{19} + 1330x^{18} + 1140x^{17} + 969x^{16} + 816x^{15} + 680x^{14} + 560x^{13} + 455x^{12} + 364x^{11} + 286x^{10} + 220x^9 + 165x^8 + 120x^7 + 84x^6 + 56x^5 + 35x^4 + 20x^3 + 10x^2 + 4x + 1), real\big)$

$$(x - 1.000000000)^4\,(x^2 + 2.85980759914041\,x + 2.09688671466619)\,(x^2 + 2.56826700779006\,x + 2.08651354409538)\,(x^2 + 2.01712323204316\,x + 2.06539425197178)\,(x^2 + 1.36383505085809\,x + 0.476897484735695)\,(x^2 + 1.26686355213634\,x + 2.03270506278077)\,(x^2 + 1.23088920992533\,x + 0.479268396234424)\,(x^2 + 0.976628665504154\,x + 0.484169063143911)\,(x^2 + 0.623240220793887\,x + 0.491955285747152)\,(x^2 + 0.400154094071160\,x \quad (6)$$

$+ 1.98697733197787) (x^2 + 0.201388353873589\, x + 0.503277004677543) (x^2 - 0.252749808868665\, x + 0.519298968164407) (x^2 - 0.486713481758057\, x + 1.92567299629874) (x^2 - 0.701283879434206\, x + 0.542242432519412) (x^2 - 1.10969728642138\, x + 0.576993916018455) (x^2 - 1.29330321158350\, x + 1.84419355629125) (x^2 - 1.45729258231815\, x + 0.638067060300958) (x^2 - 1.92323914622677\, x + 1.73312052733674) (x^2 - 2.28391758952545\, x + 1.56723338692382)$

> *# Exercice 3*

> *restart*

> $P := X^7 + 27 \cdot X^4 - X^3 - 27$

$$P := X^7 + 27 X^4 - X^3 - 27 \quad (7)$$

> *convert*$(1/P, parfrac, X)$ *# Décomposition réelle*

$$-\frac{1}{104\,(X+1)} + \frac{-X-27}{1460\,(X^2+1)} + \frac{1}{2160\,(X+3)} + \frac{163\,X - 249}{179361\,(X^2 - 3\,X + 9)} + \frac{1}{112\,(X-1)} \quad (8)$$

> *solve*(P)

$$1, -3, -1, \mathrm{I}, -\mathrm{I}, \frac{3}{2} - \frac{3\,\mathrm{I}\sqrt{3}}{2}, \frac{3}{2} + \frac{3\,\mathrm{I}\sqrt{3}}{2} \quad (9)$$

> $Q := (X-1)\cdot(X+3)\cdot(X+1)\cdot(X-I)\cdot(X+I)\cdot\left(X - \frac{3}{2} + \frac{3\cdot I\cdot \mathrm{sqrt}(3)}{2}\right)\cdot\left(X + \frac{3}{2} - \frac{3\cdot I\cdot \mathrm{sqrt}(3)}{2}\right)$

$$Q := (X-1)\,(X+3)\,(X+1)\,(X-\mathrm{I})\,(X+\mathrm{I})\left(X - \frac{3}{2} + \frac{3\,\mathrm{I}\sqrt{3}}{2}\right)\left(X + \frac{3}{2} - \frac{3\,\mathrm{I}\sqrt{3}}{2}\right) \quad (10)$$

> *convert*$\left(\frac{1}{Q}, parfrac, X\right)$ *# Décomposition complexe*

$$\frac{1}{2\,(3\,\mathrm{I}\sqrt{3}-5)\,(3\,\mathrm{I}\sqrt{3}-1)\,(X+1)} + 64 \Big/ \Big(9\,(3\,\mathrm{I}\sqrt{3}-5)\,(1+\mathrm{I}\sqrt{3})\,(3\,\mathrm{I}\sqrt{3} - 1)\,(3\,\mathrm{I}\sqrt{3}-3-2\,\mathrm{I})\,(3\,\mathrm{I}\sqrt{3}-3+2\,\mathrm{I})\,(\mathrm{I}\sqrt{3}-1)\,(-3\,\mathrm{I}\sqrt{3}+2\,X+3)\Big) \quad (11)$$

$$- \frac{1}{180\,(\mathrm{I}\sqrt{3}-3)\,(1+\mathrm{I}\sqrt{3})\,(X+3)}$$

$$+ \frac{-\frac{1}{10} + \frac{3\,\mathrm{I}}{10}}{(3\,\mathrm{I}\sqrt{3}-3-2\,\mathrm{I})\,(3\,\mathrm{I}\sqrt{3}-3+2\,\mathrm{I})\,(X+\mathrm{I})} + 64 \Big/ \Big(9\,(3\,\mathrm{I}\sqrt{3}-1)\,(\mathrm{I}\sqrt{3} - 3)\,(3\,\mathrm{I}\sqrt{3}-5)\,(3\,\mathrm{I}\sqrt{3}-3+2\,\mathrm{I})\,(3\,\mathrm{I}\sqrt{3}-3-2\,\mathrm{I})\,(\mathrm{I}\sqrt{3}-1)\,(3\,\mathrm{I}\sqrt{3}+2\,X$$

$$-3)) - \frac{\frac{1}{10}+\frac{3\,I}{10}}{\left(3\,I\sqrt{3}-3-2\,I\right)\left(3\,I\sqrt{3}-3+2\,I\right)\,(X-I)}$$

$$-\frac{1}{4\left(3\,I\sqrt{3}-5\right)\left(3\,I\sqrt{3}-1\right)\,(X-1)}$$

> *restart*

> $P := X^4 + X^2 + 1$

$$P := X^4 + X^2 + 1 \qquad \textbf{(12)}$$

> *convert*$(1/P, parfrac, X)$ *# Décomposition réelle*

$$\frac{X+1}{2\left(X^2+X+1\right)} + \frac{-X+1}{2\left(X^2-X+1\right)} \qquad \textbf{(13)}$$

> *solve*(P)

$$-\frac{1}{2}-\frac{I\sqrt{3}}{2}, -\frac{1}{2}+\frac{I\sqrt{3}}{2}, \frac{1}{2}-\frac{I\sqrt{3}}{2}, \frac{1}{2}+\frac{I\sqrt{3}}{2} \qquad \textbf{(14)}$$

> $Q := \left(X-\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\right)\cdot\left(X-\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)\right)\cdot\left(X-\left(\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\right)\cdot\left(X-\left(\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)\right)$

$$Q := \left(X+\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)\left(X+\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\left(X-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)\left(X-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right) \qquad \textbf{(15)}$$

> *convert*$(1/Q, parfrac, X)$ *# Décomposition complexe*

$$-\frac{2\,I\sqrt{3}}{\left(I\sqrt{3}+2\,X-1\right)\left(3\,I\sqrt{3}-3\right)} + \frac{2\,I\sqrt{3}}{\left(I\sqrt{3}+2\,X+1\right)\left(3\,I\sqrt{3}+3\right)} \qquad \textbf{(16)}$$

$$+\frac{2\,I\sqrt{3}}{\left(-I\sqrt{3}+2\,X+1\right)\left(3\,I\sqrt{3}-3\right)} - \frac{2\,I\sqrt{3}}{\left(-I\sqrt{3}+2\,X-1\right)\left(3\,I\sqrt{3}+3\right)}$$

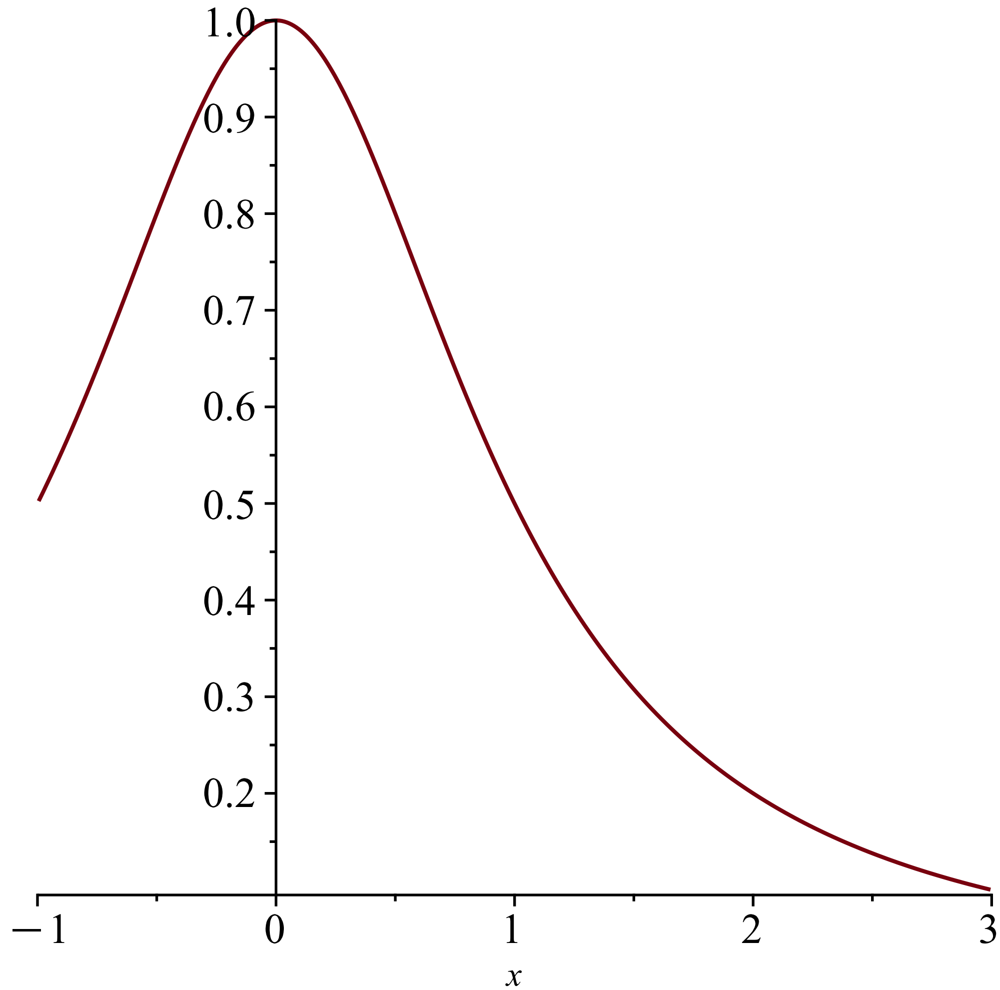
> *# Exercice 4*

> *restart*

> $f := x \rightarrow \frac{1}{1+x^2}$

$$f := x \mapsto \frac{1}{1+x^2} \qquad \textbf{(17)}$$

> *plot*$(f(x), x=-1..3)$



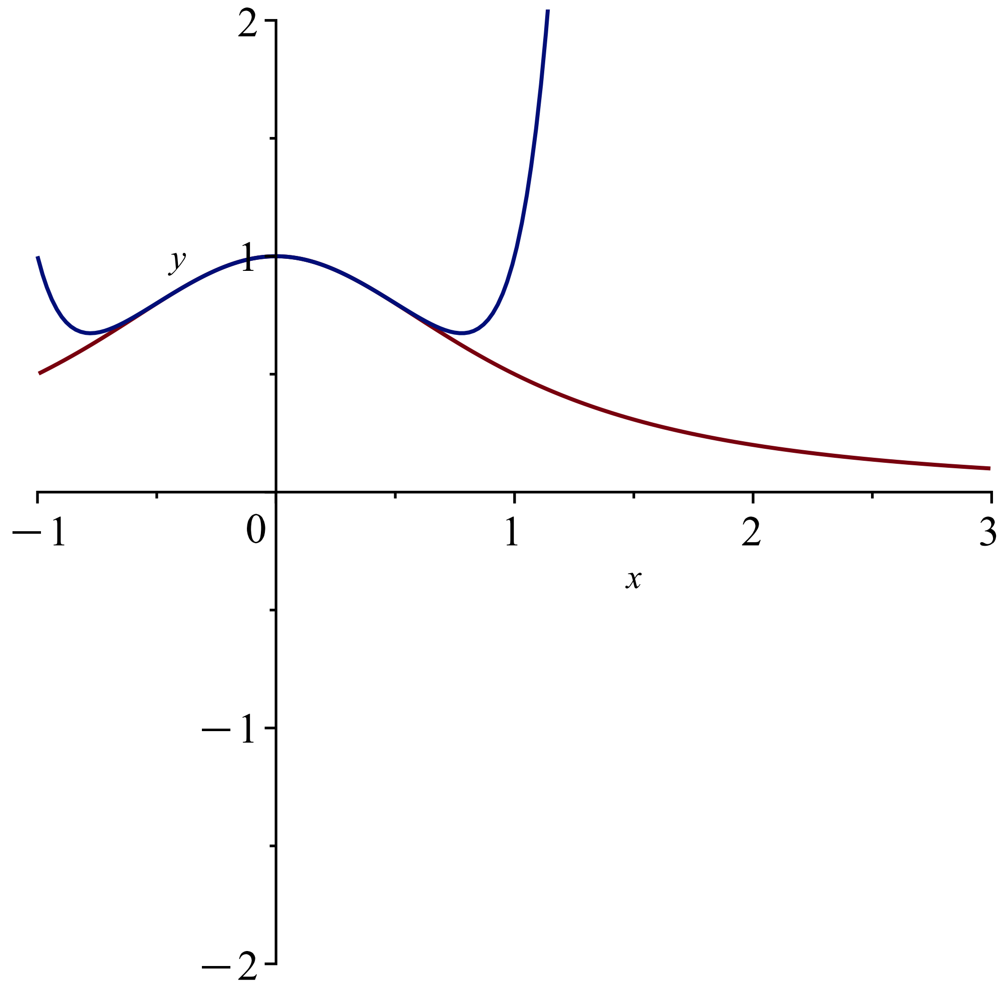
> $convert(taylor(f(x), x = 0, 10), polynom)$

$$x^8 - x^6 + x^4 - x^2 + 1 \quad \textbf{(18)}$$

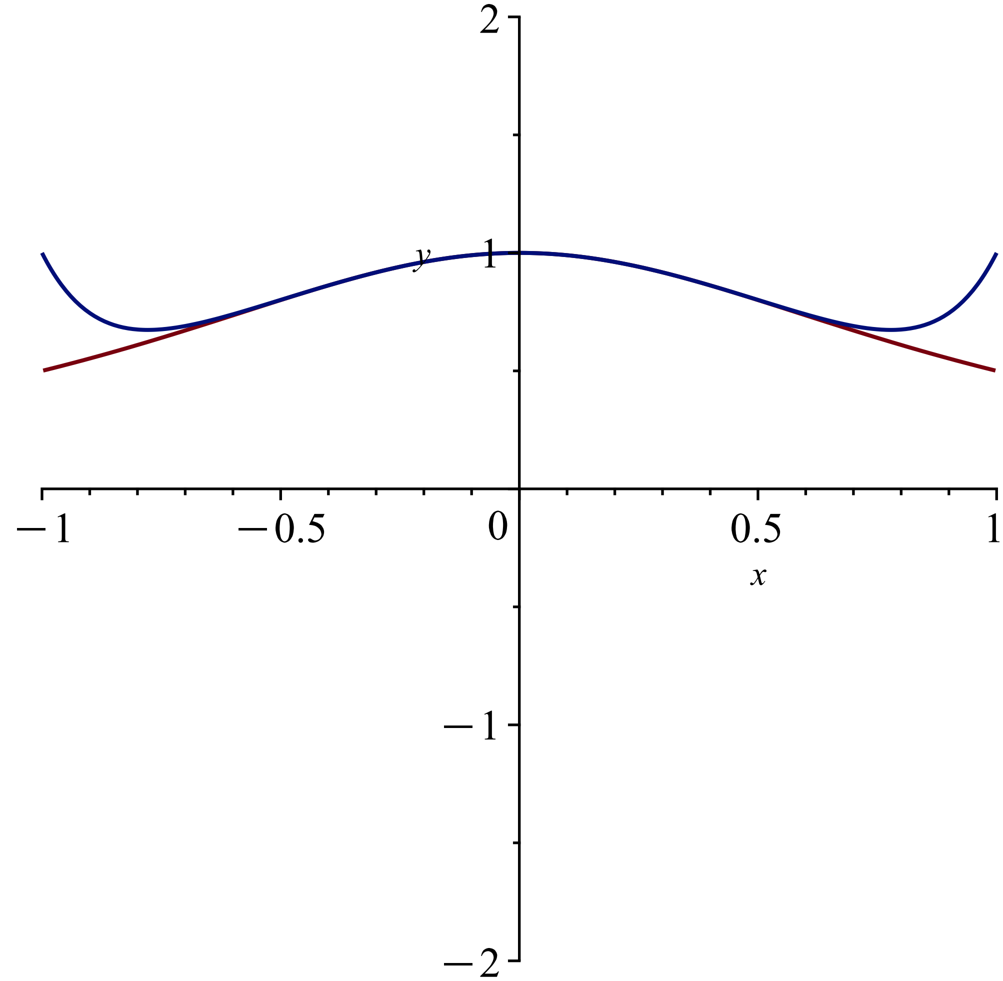
> $t := x \rightarrow x^8 - x^6 + x^4 - x^2 + 1$

$$t := x \mapsto x^8 - x^6 + x^4 - x^2 + 1 \quad \textbf{(19)}$$

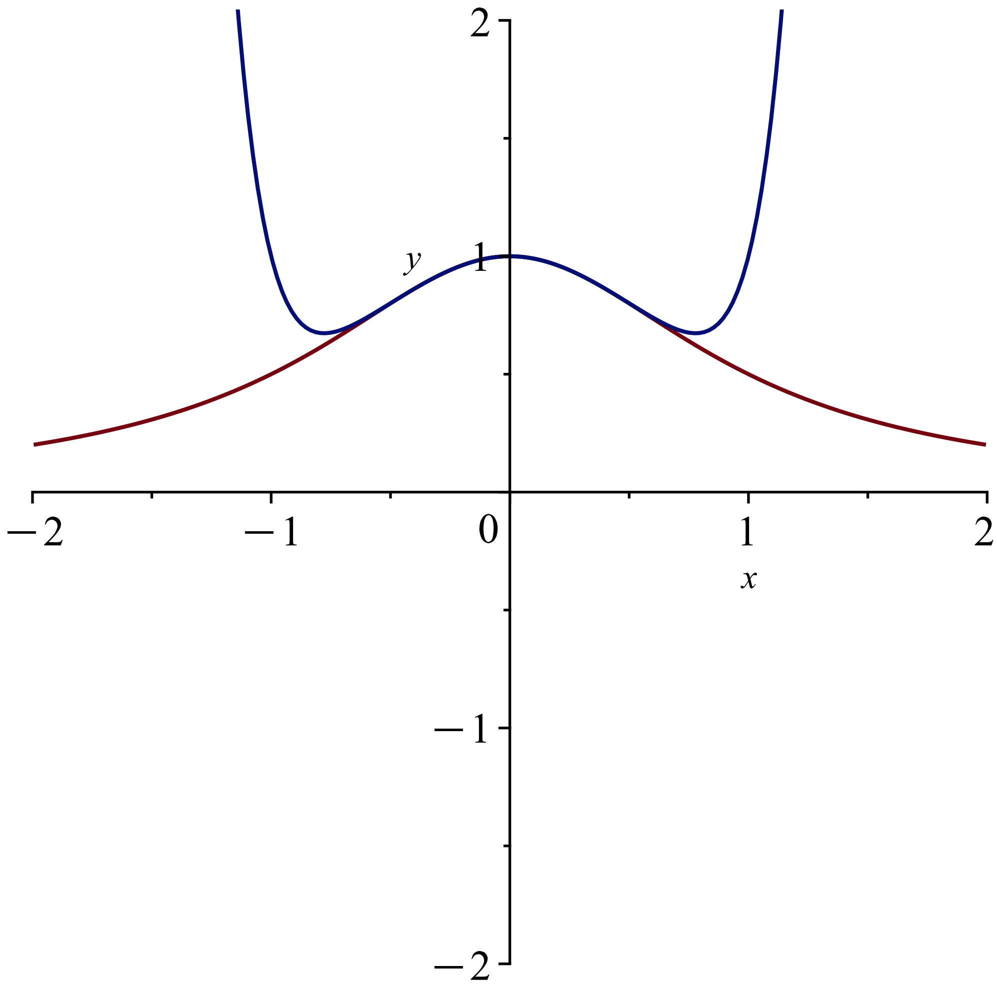
> $plot([f(x), t(x)], x = -1..3, y = -2..2)$



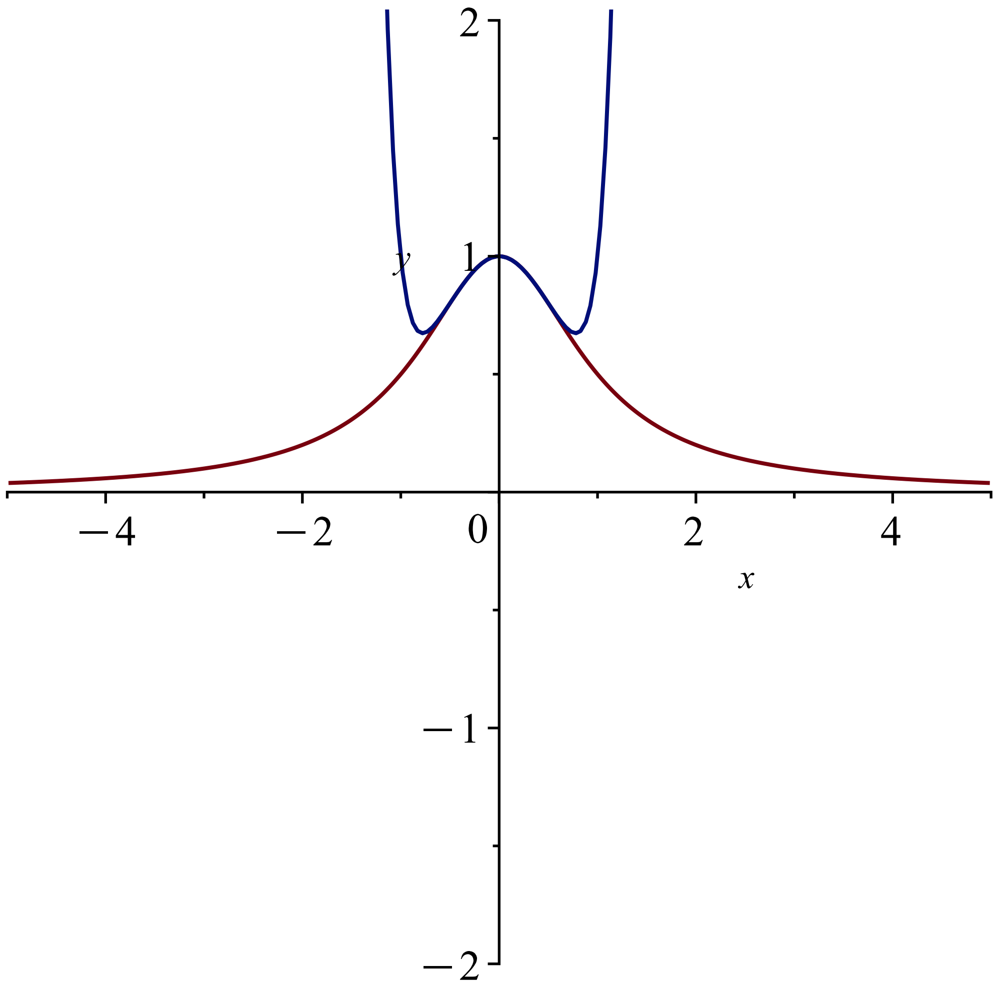
> $plot([f(x), t(x)], x=-1..1, y=-2..2)$



> $plot([f(x), t(x)], x=-2..2, y=-2..2)$



> $plot([f(x), t(x)], x=-5..5, y=-2..2)$



> $E := \left[\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}\right]; F := [seq(f(E[i]), i=1..5)]$

$$E := \left[\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}\right]$$

$$F := \left[\frac{9}{10}, \frac{9}{13}, \frac{1}{2}, \frac{9}{25}, \frac{9}{34}\right] \quad \textbf{(20)}$$

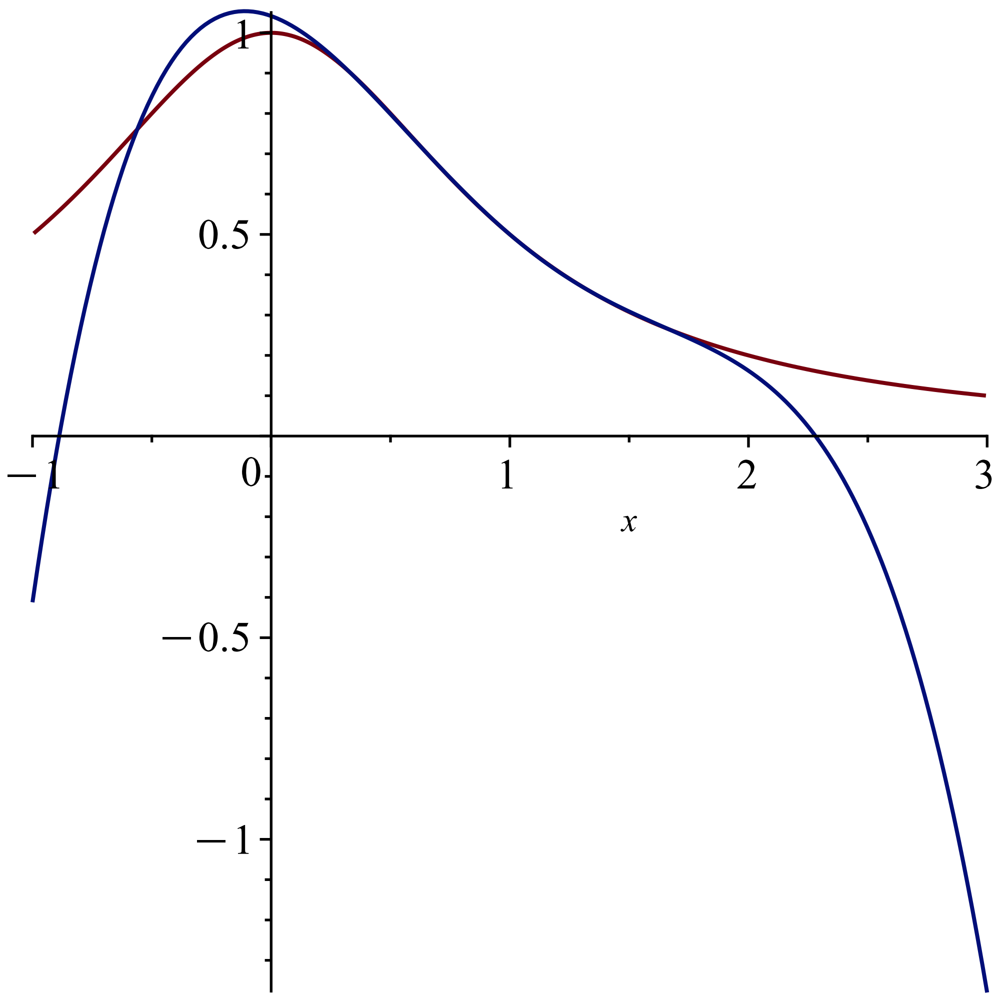
> $interp(E, F, x)$

$$-\frac{3321}{22100}x^4 + \frac{567}{850}x^3 - \frac{18729}{22100}x^2 - \frac{2331}{11050}x + \frac{1151}{1105} \quad \textbf{(21)}$$

> $p := x \rightarrow -\frac{3321}{22100}x^4 + \frac{567}{850}x^3 - \frac{18729}{22100}x^2 - \frac{2331}{11050}x + \frac{1151}{1105}$

$$p := x \mapsto -\frac{3321}{22100}\cdot x^4 + \frac{567}{850}\cdot x^3 - \frac{18729}{22100}\cdot x^2 - \frac{2331}{11050}\cdot x + \frac{1151}{1105} \quad \textbf{(22)}$$

> $plot([f(x), p(x)], x=-1..3)$



> *restart*

> $polinterp := \mathbf{proc}(f, N, a, b)$
$\quad \mathbf{local}\ i, E, F;$
$\quad E := \left[seq\left(a + i \cdot \frac{(b-a)}{N-1}, i = 0..N-1 \right) \right];$
$\quad F := [seq(f(E[i]), i = 1..N)];$
$\quad interp(E, F, x)$
$\quad \mathbf{end\ proc}:$

> $polinterp\left(x \rightarrow \frac{1}{1+x^2}, 5, \frac{1}{3}, \frac{5}{3} \right)$

$$-\frac{3321}{22100} x^4 + \frac{567}{850} x^3 - \frac{18729}{22100} x^2 - \frac{2331}{11050} x + \frac{1151}{1105} \quad \textbf{(23)}$$

> $p := polinterp\left(x \rightarrow \frac{1}{1+x^2}, 10, \frac{1}{3}, \frac{5}{3} \right)$

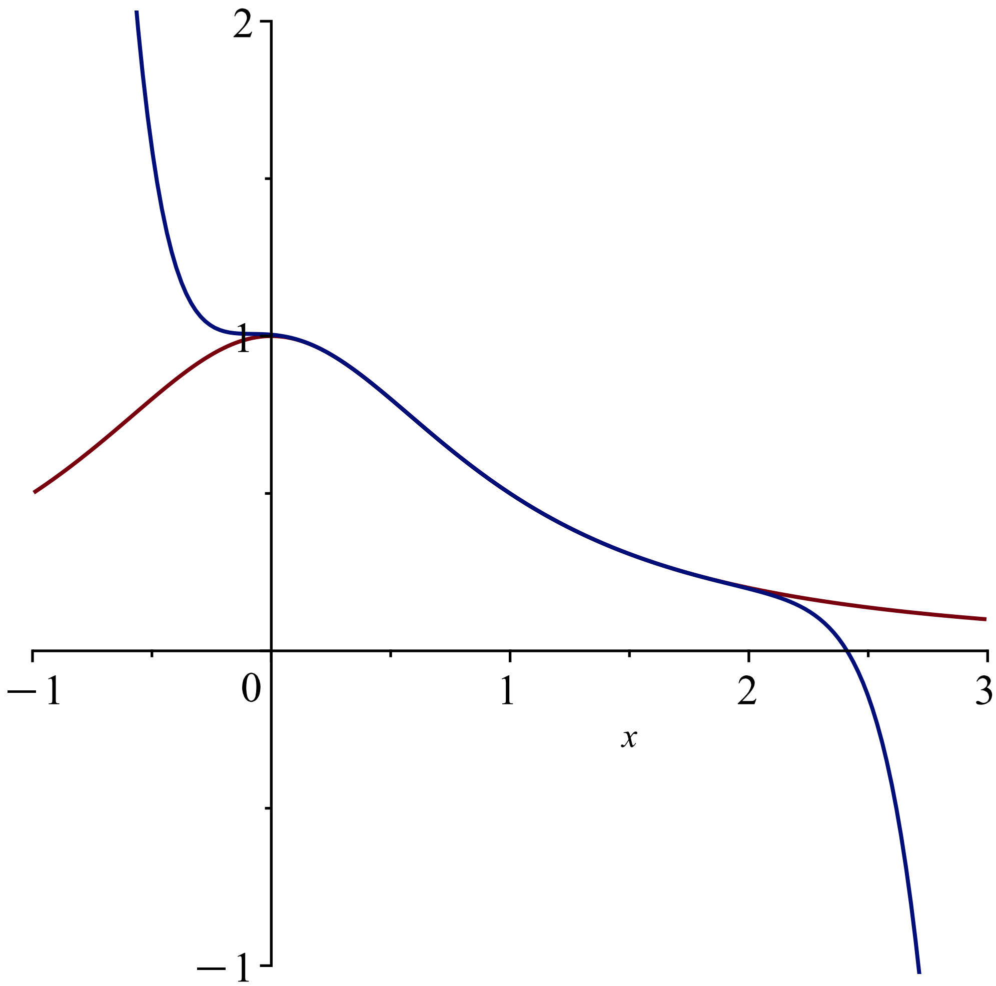
$$p := -\frac{119201756302905735033}{43864037179861090387 52} x^9 + \frac{780834614005499542200 9}{274150232374131814922 00} x^8 - \frac{713767023197943269568 3}{548300464748263629844 0} x^7 + \frac{183986628571410162582447}{5483004647482636298440 0} x^6 \quad \textbf{(24)}$$

$$- \frac{5698747981351783468359 9}{10966009294965272596880} x^5 + \frac{2508042683057236221019 77}{5483004647482636298440 0} x^4$$

$$- \frac{8476433798933038908267}{5483004647482636298440} x^3 - \frac{3196674002511928455986 7}{5483004647482636298440 0} x^2$$

$$- \frac{1409920543897049551437}{2193201858993054519376 0} x + \frac{996695719699116122 1}{992399031218576705 6}$$

> $plot\left(\left[\frac{1}{1+x^2}, p\right], x = -1..3\right)$



> $q := polinterp\left(x \to \frac{1}{1+x^2}, 20, \frac{1}{3}, \frac{5}{3}\right)$

$$q := -\frac{4367865993866108862840616232030945228697287201 1}{2057623308690226189239307532019742954385029419680 0} x - \frac{6807146833729673622591764434103364439070271290929947}{5349820602594588092022199583251331681401076491168 00000} x^{18} + \frac{2054917643040132973961015697748799114923272389}{2054733839228810674228673459966664831965078528} + \frac{8932508376221984230709488682177918095373302897596 9}{1337455150648647023005549895812832920350269122792 00000} x^{19} \quad \textbf{(25)}$$

$$- \frac{36199307587514273129767907245166556739586788970652 3}{593764772762995348726104282269848133340851996800000} x^{16}$$

$$+ \frac{3760469549942683130391347092736350580038589601096339}{3343637876621617555751387473953208230087567280698000000} x^{17}$$

$$- \frac{465912337376832807643185126500615016275277237660380 7}{302591663042680321946956990002903375644857267600000} x^{12}$$

$$+ \frac{861612226638557148447098490589968856195346512629279}{756479157606700804867392475007258439112143169000000} x^{13}$$

$$- \frac{2927443650233297984124490308927301078072086928122453}{491711452444355523163805108754717985422893059850000} x^{14}$$

$$+ \frac{4432057848007571348924837626207512566632126046501991}{1966845809777422092655220435018871941691572239400000} x^{15}$$

$$- \frac{4004531778326105916889102490759994470699182828849443}{1210366652170721287787827960011613502579429070400000} x^{10}$$

$$+ \frac{3979747328187864097797750199330711210067487550096671}{302591663042680321946956990002903375644857267600000} x^{11}$$

$$- \frac{19964465103800049809797567641244366261736476396156953}{1966845809777422092655220435018871941691572239400000} x^{7}$$

$$+ \frac{16203942029256551235787973650794837032185199606257413}{1210366652170721287787827960011613502579429070400000} x^{8}$$

$$- \frac{157380919574923652930098411261771307654782669860021}{18911978940167520121684811875181460977803579225000} x^{9}$$

$$- \frac{2332731261352166873314624865628725374486451657998431}{983422904888711046327610217509435970845786119700000} x^{5}$$

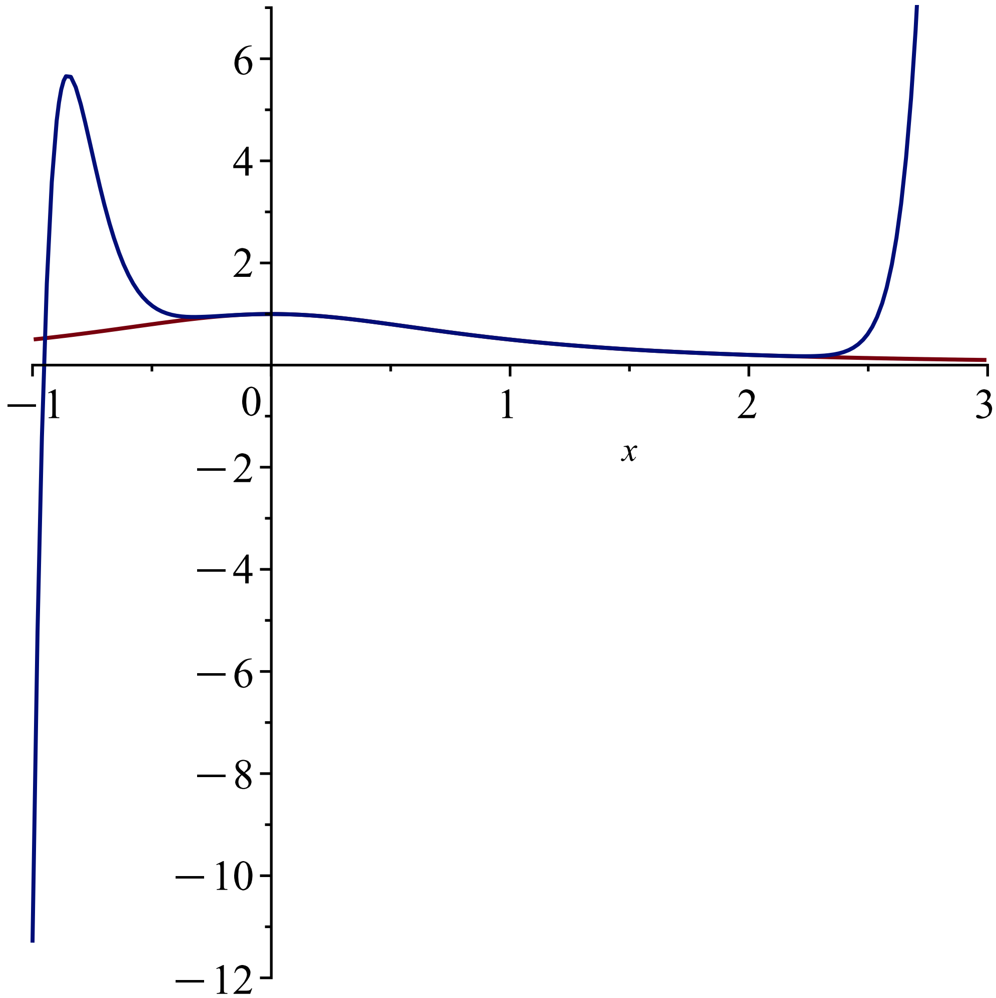
$$+ \frac{18671912010388521921688550482931106109676969563121697}{3933691619554844185310440870037743883383144478800000} x^{6}$$

$$- \frac{1444284889299438436794227902690099859428382784636 63}{92238286251630829172796544538816063472432353296000 0} x^{3}$$

$$+ \frac{994257736361000116283473898767448753932036251214 61}{578484061699241791957417775005550571085756541000 00} x^{4}$$

$$- \frac{20900785419801183159803333587646813085374420354698647}{21399282410378352368088798333005326725604305964672000} x^{2}$$

> $plot\left(\left[\frac{1}{1+x^2}, q\right], x=-1..3\right)$



> $r := polinterp\left(x \to \frac{1}{1+x^2}, 20, 0, 2\right)$

$$r := -\frac{44521358729239035114531644826116752155063468 1}{44513258479340883676902246505986685112294162 5} x^2 + \frac{189682545780375678509600487076842700 8}{402834918365075870379205850732911177486825} x + \frac{8034537085676244219954020352467216038870948}{11128314619835220919225561626496671278073540625} x^{19} + \frac{152143632572476973774536476187238653727597371 2}{11128314619835220919225561626496671278073540625} x^{17} - \frac{16231622884537901365882731445602399587052130 9}{11128314619835220919225561626496671278073540625} x^{18} + \frac{211965985296688704003184317798032088502322389504}{11128314619835220919225561626496671278073540625} x^{13} - \frac{24226883599088887125834654297115301742909650 33}{2714223078008590468103795518657724701969156 25} x^{14} + \frac{2666689215757070506064938533524789314388982912}{856024201525786224555812432807436252159503125} x^{15} \quad \textbf{(26)}$$

$$- \frac{8767093297184042021423597286069094794789986831}{1112831461983522091922556162649667127807354062 5} x^{16}$$

$$+ \frac{40847220215204703922111124705496369869074024 5504}{11128314619835220919225561626496671278073540625} x^{11}$$

$$- \frac{5577002115622824364972344157348900352818634907}{1824313872104134576922223217458470701323531 25} x^{12}$$

$$- \frac{9816446276391315806657665571422708876719613 9503}{11128314619835220919225561626496671278073540625} x^{8}$$

$$+ \frac{22339771364791577298064771788416162430220845 4656}{11128314619835220919225561626496671278073540625} x^{9}$$

$$- \frac{35920246726922666697582205988304478118616194 5617}{11128314619835220919225561626496671278073540625} x^{10}$$

$$- \frac{22597871681053191489093387039914060361496162 321}{11128314619835220919225561626496671278073540625} x^{6}$$

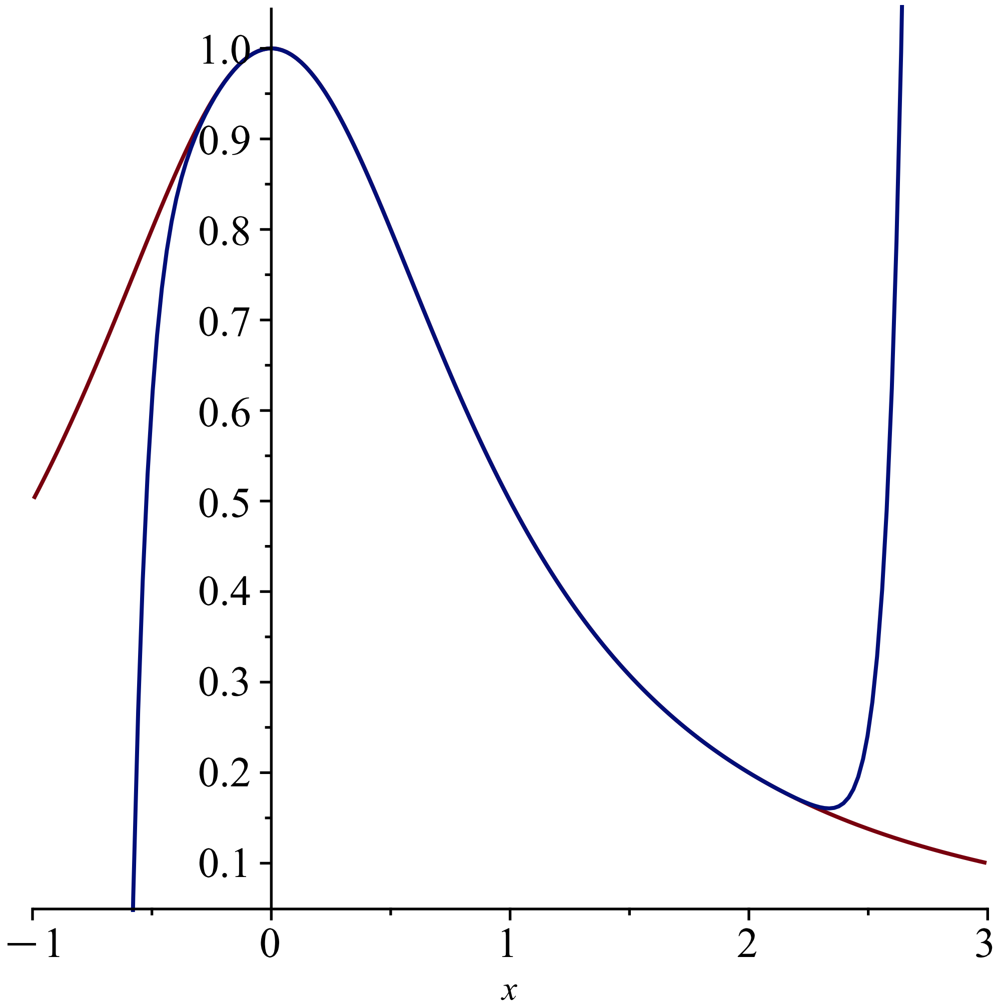
$$+ \frac{40762032465015381399651587657661712077203832 832}{11128314619835220919225561626496671278073540625} x^{7}$$

$$+ \frac{17666444278970133135303054307143163359718410 1}{18243138721041345769222232174584707013235312 5} x^{4}$$

$$+ \frac{14095431430900590766348664617876018311462912}{65848015501983555735062494831341250166115625} x^{5}$$

$$+ \frac{40788083283929669213255437568654140086681 6}{13092134846864965787324190148819613268321812 5} x^{3} + 1$$

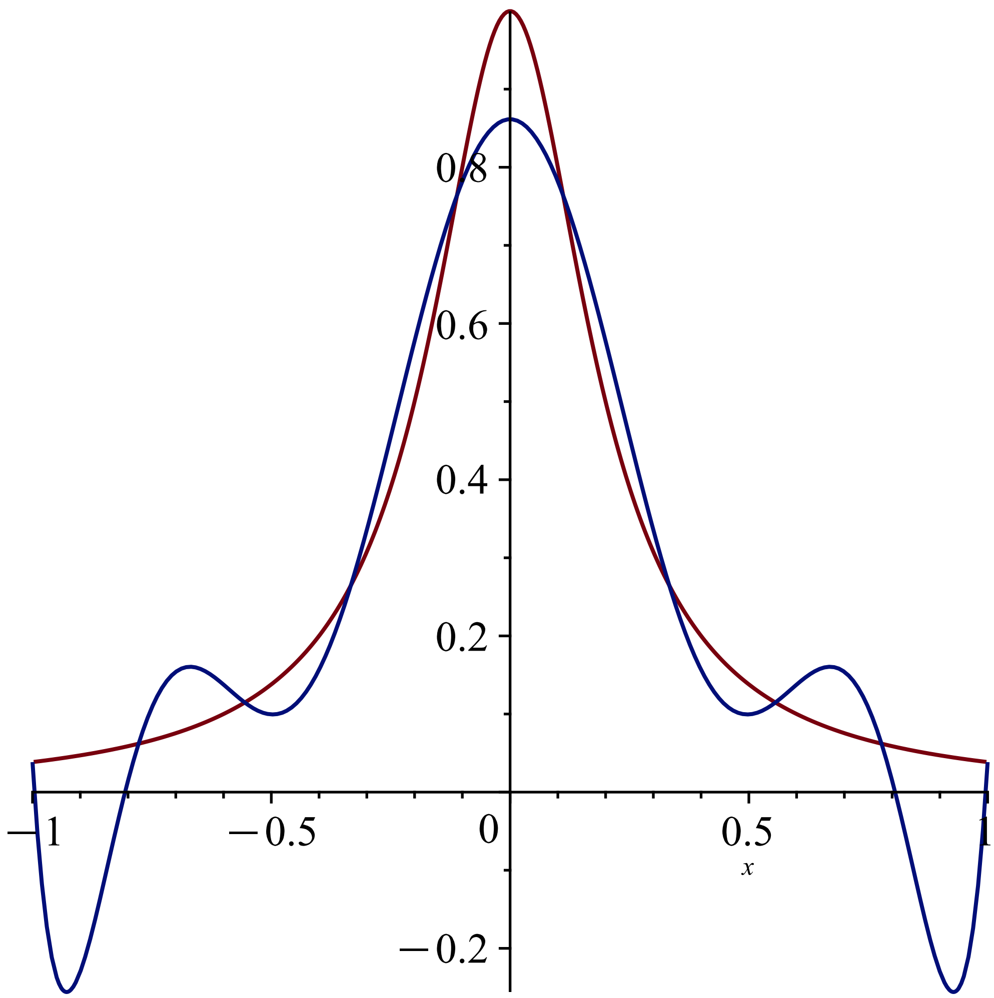
> $plot\left(\left[\frac{1}{1+x^{2}}, r\right], x=-1..3\right)$



> $s := polinterp\left(x \rightarrow \frac{1}{1 + 25 \cdot x^2}, 10, -1, 1\right);$

$$s := \frac{1868347265625}{86398461344} x^8 - \frac{1940313234375}{43199230672} x^6 + \frac{331862214375}{10799807668} x^4 - \frac{356865532425}{43199230672} x^2 + \frac{74435570719}{86398461344} \quad \textbf{(27)}$$

> $plot\left(\left[\frac{1}{1 + 25 \cdot x^2}, s\right], x = -1 \,..\, 1\right)$



> *# Exercice 5*

> *restart*

> $S := solve(x^3 + p\cdot x + q = 0, x)$

$$S := \frac{\left(-108\,q + 12\sqrt{12\,p^3 + 81\,q^2}\right)^{1/3}}{6} - \frac{2\,p}{\left(-108\,q + 12\sqrt{12\,p^3 + 81\,q^2}\right)^{1/3}},$$
$$-\frac{\left(-108\,q + 12\sqrt{12\,p^3 + 81\,q^2}\right)^{1/3}}{12} + \frac{p}{\left(-108\,q + 12\sqrt{12\,p^3 + 81\,q^2}\right)^{1/3}}$$
$$+ \frac{1}{2}\left(\mathrm{I}\sqrt{3}\left(\frac{\left(-108\,q + 12\sqrt{12\,p^3 + 81\,q^2}\right)^{1/3}}{6}\right.\right.$$
$$\left.\left.+ \frac{2\,p}{\left(-108\,q + 12\sqrt{12\,p^3 + 81\,q^2}\right)^{1/3}}\right)\right), -\frac{\left(-108\,q + 12\sqrt{12\,p^3 + 81\,q^2}\right)^{1/3}}{12}$$
$$+ \frac{p}{\left(-108\,q + 12\sqrt{12\,p^3 + 81\,q^2}\right)^{1/3}} \qquad \textbf{(28)}$$

$$- \frac{1}{2} \left(I\sqrt{3} \left(\frac{\left(-108\, q + 12\sqrt{12\, p^3 + 81\, q^2}\right)^{1/3}}{6} + \frac{2\, p}{\left(-108\, q + 12\sqrt{12\, p^3 + 81\, q^2}\right)^{1/3}} \right) \right)$$

> $simplify\left(convert\left(map\left(x \rightarrow x^n, [S]\right), `+`\right)\right)$

$$6^{-n} \left(\frac{\left(-108\, q + 12\sqrt{12\, p^3 + 81\, q^2}\right)^{2/3} - 12\, p}{\left(-108\, q + 12\sqrt{12\, p^3 + 81\, q^2}\right)^{1/3}} \right)^n + 12^{-n} \left(\frac{1}{\left(-108\, q + 12\sqrt{12\, p^3 + 81\, q^2}\right)^{1/3}} \left(I\sqrt{3} \left(-108\, q + 12\sqrt{12\, p^3 + 81\, q^2}\right)^{2/3} + 12\, I\sqrt{3}\, p - \left(-108\, q + 12\sqrt{12\, p^3 + 81\, q^2}\right)^{2/3} + 12\, p \right) \right)^n + \left(- \frac{1}{12\left(-108\, q + 12\sqrt{12\, p^3 + 81\, q^2}\right)^{1/3}} \left(I\sqrt{3} \left(-108\, q + 12\sqrt{12\, p^3 + 81\, q^2}\right)^{2/3} + 12\, I\sqrt{3}\, p + \left(-108\, q + 12\sqrt{12\, p^3 + 81\, q^2}\right)^{2/3} - 12\, p \right) \right)^n \quad \textbf{(29)}$$

> $seq\left(simplify\left(convert\left(map\left(x \rightarrow x^n, [S]\right), `+`\right)\right), n = 0..6\right)$

$$3, 0, -2\, p, -3\, q, 2\, p^2, 5\, q\, p, \frac{6\left(-2\, p^3 + 3\, q\sqrt{12\, p^3 + 81\, q^2} - 27\, q^2\right)\left(2\, p^3 - 3\, q^2\right)}{\left(-9\, q + \sqrt{12\, p^3 + 81\, q^2}\right)^2} \quad \textbf{(30)}$$

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